THE EXISTENCE OF SMOOTH ATTRACTORS OF DAMPED AND DRIVEN NONLINEAR WAVE EQUATIONS

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We consider a nonlinear wave equation of the form $u_{tt} - u_{xx} + \alpha u_t + g(u) = \gamma f(x,t)$ defined on a bounded domain $\Omega \subset \mathbf{R}^3$, with initial conditions in $\mathcal{H}^1(\Omega) \times \mathcal{L}^2(\Omega)$. The nonlinear term *g* is assumed to have polynomial growth of degree less than 5, and *f* is a smooth driving function. If solutions to this equation satisfy periodic boundary conditions on Ω, then there is a compact attractor of bounded sets of $\mathcal{H}^1(\Omega) \times \mathcal{L}^2(\Omega)$ under the dynamical system defined by this equation. This attractor lies in $\mathcal{H}^{m+1}(\Omega) \times \mathcal{H}^m(\Omega)$, where *m* depends entirely upon the smoothness of *g*. Additionally, we show that if *g* is a smooth function, then so is the attractor. These results generalize the work of other authors by establishing a series of estimates on the solution which shows that for any *t*>0, and an integrating factor $\zeta(t) = e^{\lambda t}$, then $\zeta u \in \mathcal{L}^r([0,t]; \mathcal{L}^q(\Omega))$, as long as *r* and *q* satisfy the relationship $6 \leq r = \frac{2q}{q-6} < \infty$.