Statistical analysis of models of inflation in Iceland

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Summary

Models of Icelandic inflation with unit labour costs and import prices as independent variables explained more than 80% of the variance of quarterly inflation values before 1990. After that, inflation and its variability decreased and such models explained less than 60% of the observed variations. In the quarterly macroeconomic model of the Central Bank of Iceland, inflation is represented by a Phillips curve. The fit of this model after 1994 is similar to models relying only on unit labour costs and import prices. Inflation expectations are represented by the difference between the long-term interest rate and long-term indexed rate. The present investigation found no empirical evidence that this variable contains information, relevant for inflation forecasting, and the procedure used for forecasting the expectations is not supported by the data. The best fitting inflation model was a Phillips curve where inflation expectations were represented by two constant values, before and after a change in monetary policy 2001.

Ágrip


Introduction

Iceland has a long history of high and variable inflation. Sigurdsson (1974) presents a review of the period from 1914-1974. Before 1990 quarterly models of inflation had R² of the order of 0.8-0.9 with residual standard deviation about 0.015 (Gudmundsson, 1990). The main independent variables in these models were past and present values of wages and import prices. In models where only first differences were included, the sum of the coefficients of the independent variables was close to 1 and an estimated constant was negligible and insignificant. A small but significant improvement in goodness of fit was obtained by adding error correction terms. Petursson (1998) included expectations in models of inflation.

Some of the results described here were presented at the meeting of the Icelandic Mathematical Society in Borgarnes in November 2007. I am indebted to Axel Hall and Katrín Ólafsdóttir for useful comments on the present paper.
The Central Bank of Iceland has produced inflation forecasts for almost three decades. Since 2001 inflation control is by law the main concern of the Central Bank. The policy rate has been its main instrument for this purpose. Petursson, (2007) describes the economic arguments for the monetary policy. Some comments are presented in Appendix 1. This policy implies that the Central Bank’s inflation forecasts are of great importance for the conduct of its operations and for the whole economy. They are mainly based on the Quarterly Macroeconomic Model of the Icelandic Economy, QMM, constructed by the research department of the Central Bank A description of the model and the whole data base are updated regularly and presented on the Bank’s website. (Danielsson et al., 2007). Much of the present paper will concern the modelling of inflation in the QMM.

The inflation forecasts of the Central Bank are published in its Monetary Bulletin. Since March 2007 “the baseline forecast assumes a policy interest rate path that the Bank’s staff considers optimal for attaining the inflation target. The policy rate path is chosen with the aim of bringing inflation as close as possible to 2½% within an acceptable timeframe and stabilising it close to that target afterwards. In this way, monetary policy can provide a credible anchor for inflation expectations.” (Central Bank of Iceland, 2007). Olafsson (2007) describes the economic arguments for this presentation of the baseline forecast.

According to the text of the QMM, “Consumer price inflation is given by a standard expectations-augmented Phillips curve, allowing for temporary exchange rate and wage cost shocks.” The main explanatory variables in previous models, wages or unit labour costs and a price index of imported goods and services, can be calculated in a fairly unambiguous way from data series, regularly published by Statistics Iceland and the Central Bank. This does not apply to the main variables in the “expectations-augmented Phillips curve”, i.e. inflation expectations and excess demand. Selection of observations and specifications to represent these variables are an important aspect of Phillips curve modelling. Hall and Baldursson (2008) investigated forecasting of inflation expectations and inflation by the QMM in connection with the inflation target of the Central Bank.

**Time-series properties**

The data analysed here are from the database of the Quarterly Macroeconomic Model of the Central Bank (2008) and the same notation is kept for the series. The present examination will be confined to the period from 1990 to the second quarter of 2008. The series considered here will be:

- **CPI** consumer price index
- **PM** index of import prices
- **ULCT** unit labour cost
- **INFE** inflation expectations, estimated as a difference between bond rates for nominal and indexed bonds minus 2% risk premium
- **GAPAV** excess demand, estimated from GDP, average over four quarters

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2 The time series concepts referred to in this section are explained in econometrics textbooks. Most, although not quite all, of the contents of the remaining sections can be grasped without being familiar with them.
Small letters will be used for logarithmic values; \( \Delta \) denotes first difference and subscript \( t \) quarter. INFE only extends back to 1994. GDP statistics are often subject to substantial revisions so that most estimation presented here will be confined to the period from 1994:1 to 2007:4.

Two error correction variables will be included in the present analysis:

\[
\begin{align*}
\text{u1}_t &= \text{cpi}_t - \text{ulct}_t \\
\text{u2}_t &= \text{cpi}_t - \text{pm}_t 
\end{align*}
\]

The error correction variables are scaled to have zero averages from 1994:1-2007:4.
The observed inflation from 1984-2001 could not be comfortably classified as either an I(0) or an I(1) variable (Gudmundsson, 2002). The graphs in Figure 1 do not exhibit large long term variations after 1990 and this is confirmed by Augmented Dickey Fuller tests for the presence of a unit root, presented in Table 1. The import price inflation is obviously not variance-stationary; there is a marked increase in volatility after the change in monetary policy in 2001. The outlier in Δcpi in the second quarter of 2008 will become a nuisance in statistical analyses if inflation returns to more moderate values.

\[
\text{Table 1} \\
\text{Augmented Dickey-Fuller tests for Unit root with data from 1994:1-2007:4}
\]

<table>
<thead>
<tr>
<th>series</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δcpi</td>
<td>0.011</td>
</tr>
<tr>
<td>Δpm</td>
<td>0.000</td>
</tr>
<tr>
<td>Δulct</td>
<td>0.000</td>
</tr>
<tr>
<td>INFE</td>
<td>0.052</td>
</tr>
<tr>
<td>u1</td>
<td>0.614</td>
</tr>
<tr>
<td>u2</td>
<td>0.061</td>
</tr>
</tbody>
</table>

Error correction or cointegration, involving consumer prices, wage costs and import prices, have figured prominently in models of inflation in Iceland and elsewhere. According to Figure 2 there is no indication of a permanent imbalance between consumer prices and import prices. But the ratio between consumer prices and unit labour costs has decreased by about 0.2 on a log-scale from the first years in the series to the present time. The unit root hypothesis is accepted for u1, even if a linear trend is included in the model. This could be caused by a systematic long term error in the estimation of the unit labour cost relevant to the consumer price index. It is harmless for the QMM inflation model where ulct only appears as a first difference. But in models with error correction terms including this series, it could prevent detection of actual relationships or introduce spurious relationships.

Indirect taxation on food was reduced in the first quarter of 2007. There is no indication of exceptional negative residuals in inflation models, but if the effect on prices is spread over a longer period it would affect the error correction terms.
Models of inflation

The Phillips curve model of consumer price inflation in the QMM is

\[ \Delta \text{cpi}_t = 0.741 \text{INFE}_t/4 + 0.059 \Delta \text{pm}_t + 0.080 \Delta \text{pm}_{t-1} + 0.120 \Delta \text{ulc}_t - 0.120 \Delta \text{GAPAV}_{t-1}, \]

(11.7) (1.9) (2.5) (3.6)

\[ R^2 = 0.548; \ s=0.0046. \quad \text{(QMM, equation 7.1)} \]

The parameters were estimated with observations from 1994-2004. The values in parentheses are t-values, i.e. the ratio between the numerical value of the parameter and its estimated standard deviation. Homogeneity is imposed. This implies that the coefficients of INFE$_t/4$, $\Delta \text{pm}_t$, $\Delta \text{pm}_{t-1}$ and $\Delta \text{ulc}_{t-1}$ are constrained to add up to 1. The constraint passes tests and “ensures a vertical long run Phillips curve and hence, that no long term trade off between inflation and output exists.”

Let us now repeat this estimation with data up to 2007:4 from the data set published by the Central bank in July 2008. As inflationary expectations of bond traders might be influenced by current inflation I use two stage least squares, with INFE$_t$, $\Delta \text{cpi}_t$ and a constant as additional instruments. Regression of INFE$_t$ on the instruments produces $R^2$ about 0.8. In this formulation of an inflation model, there is negligible difference between results from the ordinary least squares and the two stage least squares.

\[ \Delta \text{cpi}_t = 0.766 \text{INFE}_t/4 + 0.062 \Delta \text{pm}_t + 0.056 \Delta \text{pm}_{t-1} + 0.116 \Delta \text{ulc}_{t-1} + 0.096 \text{GAPAV}_{t-1}, \]

(15.0) (3.2) (2.9) (2.7) (3.7)

\[ R^2 = 0.586; \ s = 0.00481. \]

The equation passes tests on normality and absence of serial correlation in residuals but fails on heteroskedasticity. These tests were performed with other equations presented here, but the test statistics are uninteresting unless important premises of the estimation are rejected and will not be reported. The changes in parameter values from the QMM model are negligible for economic interpretation.

In the QMM model, the inflation is mainly determined by inflation expectations. It differs markedly from earlier models where wages and import prices were the main explanatory variables. Estimation of such model for 1994-2007 gives

\[ \Delta \text{cpi}_t = 0.0064 + 0.085 \Delta \text{pm}_t + 0.064 \Delta \text{pm}_{t-1} + 0.095 \Delta \text{ulc}_{t-1} - 0.047 \text{u1}_{t-1} - 0.015 \text{u2}_{t-1}, \]

(7.3) (4.2) (3.0) (2.1) (4.7) (1.0)

\[ R^2 = 0.596; \ s = 0.00473. \]

This model will be referred to as the Old Model in the following text. The goodness of fit differs little from the QMM model, and parameters of the first differences of import prices and unit labour costs are fairly similar. $R^2$ is considerably lower than in models
estimated with data before 1990, but the standard deviation of the residuals is lower because the variability of inflation is much lower. The constant term is an important feature of this model and accounts for a quarterly inflation of 0.64\%, or 2.6\% annually.

The presence of $u_{1,t-1}$ with a respectable t-value in equation (2) is anomalous from the point of view of time series analysis. As pointed out in the previous section, this variable is clearly not acceptable as an I(0) variable but all other variables are, including the dependent variable. The t-value of $u_{1,t-1}$ drops just below 2 when a time trend is also included, but as the trend variable has a very low t-value the present form is preferable.

Let us now produce a model that contains both the QMM and the traditional models as special cases by including each variable that appears on the right hand side of either of them without any constraints on the parameters. Estimation with instrumental variables for INFE gives

$$
\Delta \text{cpi}_t = 0.0048 + 0.082 \Delta \text{pm}_t + 0.053 \Delta \text{pm}_{t-1} + 0.092 \Delta \text{ucl}_{t-1} - 0.030 u_{1,t-1} - 0.020 u_{2,t-1} \\
(1.7) \quad (3.9) \quad (2.4) \quad (2.1) \quad (2.1) \quad (1.3) \\
+ 0.188 \text{INFE}_t/4 + 0.068 \text{GAPAV}_{t-1}, \quad (3)
$$

$$R^2 = 0.603; \quad s = 0.00469.$$

Each of the models in equations (1) and (2) can be regarded as a simplified version of the more general model in equation (3), obtained by estimation after imposing linear constraints on the parameters. (The constraints consist of assigning zero value to the coefficients of variables which only appear in the other model. In order to produce the model from the QMM, the homogeneity condition must also be imposed). The validity of the simplifications can be tested and both are acceptable; the QMM with a p-value of 0.17 and the Old Model with a p-value of 0.25.

A simple way to compare two models of the same dependent variable, where neither is a special case of the other, is based on the instrumental variables technique. We estimate $\Delta \text{cpi}_t$ by equations (1) and (2) and call the estimated series $\Delta \text{cpiqmm}_t$ and $\Delta \text{cpiold}_t$. The QMM model is then reestimated with $\Delta \text{cpiold}_t$ as an additional variable. If $\Delta \text{cpiold}_t$ gets a significant coefficient it is an indication that the Old Model contains information that is lacking in the QMM and vice versa with $\Delta \text{cpiqmm}_t$.

The augmented QMM model is:

$$
\Delta \text{cpi}_t = 0.254 \text{INFE}_t/4 + 0.018 \Delta \text{pm}_t + 0.016 \Delta \text{pm}_{t-1} + 0.029 \Delta \text{ucl}_{t-1} + 0.049 \text{GAPAV}_{t-1} \\
(1.2) \quad (0.7) \quad (0.6) \quad (0.5) \quad (1.5) \\
+ 0.684 \Delta \text{cpiold}_t, \quad (4) \\
(R^2 = 0.622; \quad s = 0.00457).$$
The Old Model, augmented by inflation estimated by the QMM is

\[
\Delta \text{cpi}_t = 0.0036 + 0.049 \Delta \text{pmt}_t + 0.026 \Delta \text{pmt}_{t-1} + 0.035 \Delta \text{uclt}_{t-1} - 0.032 \text{u1}_{t-1} - 0.019 \text{u2}_{t-1} + 0.480 \Delta \text{cpiqmm}_t,
\]

\[
(1.8) \quad (1.6) \quad (0.8) \quad (0.6) \quad (2.4) \quad (1.3)
\]

\[
(R^2 = 0.608; \quad s = 0.00466).
\]

(R\(^2\) and s are calculated without regard to the degrees of freedom used in calculating \(\Delta \text{cpiold}\) and \(\Delta \text{cpiqmm}\)). The results for the QMM model were obtained by maintaining the restriction that coefficients of variables other than GAPAV add up to one, but the results with an unrestricted model are very similar.

The t-value of \(\Delta \text{cpiold}\) when it is included in the QMM model indicates that information about wages and import prices may not be fully utilized in the QMM. However, the improvement in goodness of fit is not large enough to make much practical difference. The coefficient of \(\Delta \text{cpiqmm}\) is not highly significant which is in accordance with low t-values of INFE and GAPAV in equation (3).

This instrumental variable procedure is related to the J-test for non-nested hypotheses by Davidson and MacKinnon (1981), described in Appendix 2. In that test the null hypothesis that the QMM model is true gets p-value 0.09 and the Old Model gets p-value 0.24.

**Inflation expectations**

The QMM model of Icelandic inflation is selected by economic considerations. In view of the importance attached to inflation expectations by the Central Bank, it is uncomfortable that the coefficient of the interest rate difference, which the authors of the QMM have chosen to represent this concept, drops to insignificance in the expanded versions of equations (3) and (4). However, this may be an indication that the interest rate difference is an inadequate representation of inflation expectations rather than that the Phillips curve model is at fault. There is a speculative element in bond rates that might differ from inflation expectations, or the expectations of bond traders might differ from other economic agents with a stronger influence on the inflation.

In the present section we examine some alternative expressions for inflation expectations in the Phillips curve model of the QMM, retaining the homogeneity condition. The name INFE is kept for the observed series from the QMM.

An expected long term average is a common prediction, applied in various subjects and circumstances and might express the expectations of some agents better than INFE. So let us represent inflation expectations in the Phillips curve model by a weighted average of
constant expectations, Δcpi0, and INFE. The value of Δcpi0 can be estimated because of the homogeneity condition. This gives the following results:

$$Δcpi_t = 0.756[0.408\text{INFE}_t/4 + 0.592Δcpi0] + 0.069Δpm_t + 0.061Δpm_{t-1} + 0.113Δuclt_{t-1} + 0.117\text{GAPAV}_{t-1},$$

$$R^2 = 0.578; \quad s = 0.00483; \quad Δcpi0 = 0.0079, \quad t = 5.0.$$  

The correlation coefficient between the estimated parameters of INFE$_t/4$ and Δcpi0 is -0.991 and neither is significantly estimated. In fact the goodness of fit is practically the same whether we use either Δcpi0 or INFE$_t/4$ alone or the linear combination above.

I suspect that inflation expectations of some important agents in the Icelandic economy are strongly affected by comparison of the consumer prices, wage level and import prices. When u1 and u2 are included in the expression for expectations, together with INFE and Δcpi0, the t-value of the estimated coefficient of INFE$_t$ is 0.5. For practical purposes variables with t-values below 1 are usually best left out. Estimation without INFE gives

$$Δcpi_t = 0.765 \left[0.0081 - 0.040u_{1,t-1} - 0.026u_{2,t-1}\right] + 0.085Δpm_t + 0.056Δpm_{t-1} + 0.094Δuclt_{t-1} + 0.073\text{GAPAV}_{t-1},$$

$$R^2 = 0.608; \quad s = 0.00465.$$  

Unlike INFE$_t$ the error correction terms appear to convey some information about inflation. However, the fact that u1$_t$ exhibits much stronger long term variations than the other variables implies that interpretation of this is rather uncertain. It seems likely that long term variations in wages or productivity are not correctly represented in u1. The variable u1 might act as a substitute for some long term variations affecting inflation but inadequately accounted for by the variables included in the model.

Let us now include monetary policy in the set of explanatory variables for inflation expectations and define

$$\text{TARGET}_t = 0 \text{ for 1994:1-2001:1},$$

$$\text{TARGET}_t = 1 \text{ for 2001:2-2007:4}.$$  

When this variable is included in the model for inflation expectations, INFE and the error correction terms become insignificant and the estimated Phillips curve is
\[ \Delta \text{cpi}_t = 0.742 [0.0056 + 0.0048 \text{TARGET}_t] + 0.078 \Delta \text{pm}_t + 0.067 \Delta \text{pm}_{t-1} \]
\[ + 0.113 \Delta \text{ucl}_{t-1} + 0.099 \text{GAPAV}_{t-1}, \]
\[ R^2 = 0.625; \quad s = 0.00456. \]

According to this equation, inflation expectations increased from 2.3% to 4.2% when the inflation target, 2.5%, was adopted by the Central Bank. The correlation coefficient between the estimated values of the constant term in the inflation expectations and the parameter of the TARGET variable is -0.44 so that the standard deviation of the inflation expectations is similar in both periods.

Test of the null hypothesis that the QMM Phillips curve model is true against the alternative model of equation (6) by the J-test of Davidson and MacKinnon (1981) (Appendix 2) leads to rejection. The parameter \( a \) is estimated as 0.82 with \( t \)-value 2.9 which implies rejection with \( p = 0.005 \). Test of the null hypothesis that equation (6) is the true model against the alternative model from the QMM is accepted with \( p = 0.31 \).

Another approach to inflation expectations in the QMM Phillips curve is to define them as a state variable and estimate them by the Kalman filter. The model can be expressed as

\[ \Delta \text{cpi}_t = \theta_0 y_t + \sum_{j=1}^{J} \theta_j z_{jt} + \varepsilon_t, \]
\[ y_t = \varphi_0 y_{t-1} + \sum_{j=1}^{J} \varphi_j x_{jt} + \delta_t, \]

where \( y_t \) is the unobserved state variable, representing inflation expectations, \( z_{jt} \) are the remaining variables in the Phillips curve model and \( x_{jt} \) are variables which could provide information about the inflation expectations. Residuals, \( \delta_t \), are now included in the definition of the inflation expectations as well as the residuals \( \varepsilon_t \) in the Phillips curve.

As the series employed in our estimations only extend to 56 observations, the possibilities of estimating models of this kind are limited. If the error correction terms or the TARGET dummy series are included, the estimated variance of \( \delta_t \) becomes zero and \( \varphi_0 = 0 \) so that the model is reduced to equations (5) or (6). If all independent variables are left out of the state space model, \( \varphi_0 \) is estimated as 0.33, but as the standard deviation of this value is 0.35 we get little information about the time series properties of inflation expectations by this methodology. However, random walk is rejected; if \( \varphi_0 \) is fixed as 1 the variance of \( \delta_t \) is estimated as zero so that the inflation expectations become constant.
Inflation forecasting

With a coefficient of 0.74 in the model for $\Delta \text{cpi}_t$ in the QMM, inflation expectations have a dominating influence on forecasts of inflation. Future values of $\text{INFE}_t$ are obtained by the equation

$$\text{INFE}_t = 0.3 \text{INFE}_{t-1} + 0.3 \text{INF}_{t-1} + 0.4 \text{IT},$$

QMM equation (7.17)

where $\text{INF}_t$ is four quarter CPI inflation rate and IT the Central Bank of Iceland 2.5% inflation target.

This is not an estimated equation but presents the opinion of the authors of the QMM about how inflation expectations will proceed after publication of an inflation forecast by the Central Bank. The text of the QMM (Danielsson et al., 2007) provides no information about how it was selected beyond the general information that there is “close likeness of QMM and the Medium-term Macro Model (MTMM) of the Bank of England.” Hall and Baldursson (2008) compare the treatment of inflation expectations in these models.

Estimation of the parameters in the QMM equation (7.17) with observations from the second quarter 2001, after the Central Bank adopted an inflation target, to fourth quarter 2007, keeping the constraint that they sum to one, gives

$$\text{INFE}_t = 1.040 \text{INFE}_{t-1} - 0.066 \text{INF}_{t-1} + 0.026 \text{IT},$$

(7)

$$R^2 = 0.737; \quad s = 0.00521.$$

It is obvious from this that the series $\text{INFE}_t$, used in estimating the Phillips curve model in the QMM, does not follow the development of expectations assumed by the authors of the QMM in equation (7.17). However, our investigation suggests that the interest rate difference is not a good indicator of expectations, affecting Icelandic inflation. The estimated parameters in equation (7) might therefore not provide a strong argument against the application of equation (7.17) for forecasting. But the alternative representations of inflation expectations, examined in the previous section, offer no support for the QMM equation (7.17) for forecasting the expectations. Equation (6) entails a direct rejection of the assumption that expectations converge towards the inflation target of the Central Bank.

Discussion

The models examined here explain about 60% of the variance of quarterly observations of inflation from 1994 to 2007, compared to 80% or more before 1990. While annual inflation was of the order of 20%, the increases in wages and import prices were so large that sellers had to adjust their prices to the costs when respective product was sold. With lower cost increases, sellers had more scope to choose when they were added to the price.
After 1990, inflation became of the same order as productivity changes. They are probably less accurately observed than wages and prices and with variable time lags in their relationship with inflation. Their magnitude was presumably similar before and after 1990 so that the relative impact on variations in inflation has increased.

The stochastic element in the inflation models presented and estimated here has the form of residuals, added to the right hand side of the equations. This is a normal model of measurement errors of the dependent variable. It can also represent external disturbances, uncorrelated with the independent variables included in the model. But it is not a correct description of measurement errors in independent variables or the effects of irregular timing of the impacts of observed shocks. Irregular timing can be modelled theoretically, but estimation is difficult (Gudmundsson, 1998).

The combined effects of the Phillips curve model and the forecasting model of expectations in the QMM are crucial for producing a baseline forecast where the inflation target of the Central Bank of Iceland is reached within an acceptable timeframe. (Hall and Baldursson, 2008). In the present investigation, any extension of the QMM Phillips curve, allowing for other representations of expectations than the interest rate difference, results in a small and insignificant parameter for this variable. The properties of the forecasting model for expectations are widely different from the observed properties of the interest rate difference. They are also incompatible with other models of expectations examined here.

The result that inflation expectations became higher after inflation control with a given target became officially the main goal of monetary policy calls for an explanation or refutation. Here are some comments:

The simple explanation, that not only monetary policy, but various other things that may affect inflation were different in 2001-2007 from what they were in 1994-2000, is inadequate. The Phillips curve model of the QMM contains a carefully chosen demand indicator. It also “allows for a temporary exchange rate and labour cost shocks”. I put the error correction variables representing imbalance between inflation and labour costs and import prices at its disposal, but they were found redundant. Together all these variables should be better suited than a constant addition to expectations during the period of the inflation target to account for the demand enhancing activity of the government and commercial banks, oil price increases and the effects of exceptional investments in power stations and aluminium factories.

The commitment of the Central Bank to the task of controlling inflation has gained a lot of publicity. This may have reduced the concern of other agents in the economy, such as government, trade unions and employers, about inflation; the Central Bank would take care of it. But in spite of the increased emphasis on the Central Bank’s role in combating inflation, it received no additional tools for achieving this goal. On the contrary, it gave up its commitment to keep the exchange rate within prescribed bounds because Iceland could not afford it.
Several representations of inflation expectations with widely different economic contents have been estimated in the context of the Phillips curve model from the QMM. But their weight in the model is always practically the same, about 0.75. None of the models fits the data substantially better than the simple model of constant inflation expectations, although the difference between some of the models is statistically significant. In view of this, I doubt that this modelling and estimation provides a valid description of the quantitative impact of inflation expectations.

References


Appendix 1.

Comments on monetary policy

The main goal of the present monetary policy in Iceland is to keep annual inflation close to a target of 2.5%. The Central Bank uses its policy rate as the sole control instrument for this purpose.

The policy rate is a powerful control instrument, affecting many important economic variables. Considering the multifarious interconnections in the economy, I am sceptical of the policy to apply this instrument only with regard to one variable, which is affected by various other potential control instruments. The following is a simple sketch of the basis for this scepticism, leaving out many details and qualifications.

Economic policy aims to maximize a utility function, \( U(y) \), where \( y \) is a vector of relevant economic variables. Let \( y_1 \) be inflation.

The authorities have instruments, \( x \), to control the variables in \( U \). Let \( x_1 \) be the policy rate. We assume that all relevant derivatives exist. The effect on \( U \) from changing \( x_1 \) by \( \Delta x_1 \) is then

\[
\Delta U = \sum_j \frac{\partial U}{\partial y_j} \frac{\partial y_j}{\partial x_1} \Delta x_1.
\]

If \( y_1 \) is not on target, \( U \) will increase by changing \( y_1 \) in that direction. The relationship between \( y_1 \) and \( x_1 \) is known, (or at least the sign of the first derivative), so that \( \Delta x_1 \) can be applied to move \( y_1 \) towards the target.

However, \( x_1 \) affects more \( y_j \) than \( y_1 \) so that \( U \) may not increase by using \( \Delta x_1 \) to move \( y_1 \) in the right direction. And \( x_1 \) is not the only control instrument affecting \( y_1 \) so it is not immediately obvious that it is always the best instrument for steering inflation.

Appendix 2.

\( J \)-test from Davidson and MacKinnock

Two non-nested, possibly non-linear, models for \( y_t \) are

\[
y_t = g_t(X_{1t}; \alpha) + \varepsilon_{1t}
\]

and
\[ y_t = g_2(X_{2t}; \beta) + \epsilon_{2t}, \]

where \( X_{1t} \) and \( X_{2t} \) are vectors of independent variables and \( \alpha \) and \( \beta \) vectors of parameters and \( \epsilon_{1t} \) and \( \epsilon_{2t} \) residuals. Suppose we want to test the null hypothesis that \( g_1 \) is the true model of \( y_t \). The alternative model \( g_2 \) is estimated by maximum likelihood (or least squares if appropriate). The estimated values are \( g_2(X_{2t}; \hat{\beta}) \). Estimate also

\[ y_t = (1-a) \, g_1(X_{1t}; \alpha) + a \, g_2(X_{2t}; \hat{\beta}) + \epsilon_{1t}. \]

When \( H_0 \) is true, the parameter \( a \) is zero so that a significant value of \( a \) implies rejection of the null hypothesis. Rejection of the null does not imply that the alternative model is true, but the test has no power unless \( g_2 \) contains some information about \( y_t \) which is lacking in \( g_1 \). The roles of \( g_1 \) and \( g_2 \) can be reversed.